

Density perturbations in the gas of wormholes

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Abstract

The observed dark matter phenomenon is attributed to the presence of a gas of wormholes. We show that due to topological polarization effects the background density of baryons generates non-vanishing values for wormhole rest masses. We infer basic formulas for the scattering section between baryons and wormholes and equations of motion. Such equations are then used for the kinetic and hydrodynamic description of the gas of wormholes. In the Newtonian approximation we consider the behavior of density perturbations and show that at very large distances wormholes behave exactly like heavy non-baryon particles, thus reproducing all features of CDM models. At smaller scales (at galaxies) wormholes strongly interact with baryons and cure the problem of cusps. We also show that collisions of wormholes and baryons lead to some additional damping of the Jeans instability in baryons.

1 Introduction

The nature of Dark Matter (DM) represents one of the most important and yet unsolved problems of the modern astrophysics. Indeed, while the presence of DM has long been known [1] and represents a well established fact (e.g., see [2, 3] and references therein), there is no common agreement about what DM is. In the simplest picture DM represents some non-baryonic particles (predicted numerously by particle physics) which should be sufficiently heavy to be cold at the moment of recombination and those give the basis to the standard (cold dark matter) CDM models. The latter turn out to be very successful in reproducing properties of the Universe at very large scales (where perturbations are still on the linear stage of the development) which led to a wide-spread optimistic believe that non-baryonic particles provide indeed an adequate content of DM.

However the success of CDM models at very large scales is accompanied with a failure at smaller (of the galaxies size) scales. Indeed, cold particles which interact only by gravity should necessary form cusps ($\rho_{DM} \sim 1/r$) in centers of galaxies¹ [4] (see also [5] where the problem of cusps in CDM is discussed

¹The presence of cusps formed by the development of adiabatic perturbations follows straightforwardly from the conservation of the circulation theorem in the hydrodynamics. By

in more detail), while observations [6] definitely show the cored ($\rho_{DM} \sim const$) distribution. The only way to destroy the cusp and get the cored distribution is to introduce some self-interaction in DM or to consider warm DM. Both possibilities are rejected at large scales by observing $\Delta T/T$ spectrum (e.g., see [3] and references therein). By other words DM displays so non-trivial properties (it is warm or self-interacting in galaxies, however it was cold at the moment of recombination and it is still cold on larger (than galaxies) scales) that it is difficult to find particles capable of reconciling such observations.

These facts support the constant interest to different alternatives of the DM hypothesis which interpret the observed discrepancy between luminous and gravitational masses as a violation of the law of gravity. Such violations (or modifications of general relativity (GR)) have widely been discussed, e.g., see [7, 8]. However, it turns out to be rather difficult to get a modification of GR which is flexible enough to reconcile all the variety of the observed DM halos. Moreover, the weak lensing observations of a cluster merge [9] seem to reject most of modifications of GR in which a non-standard gravity force scales with baryonic mass.

The more viable picture of DM phenomena was suggested by [10] (see also references therein) and developed recently by [13] [16]. It is based on the fact that on the very early (quantum) stage the Universe should have a foam-like topological structure [17]. There are no convincing theoretical arguments of why such a foamed structure should decay upon the quantum stage - relics of the quantum stage foam might very well survive the cosmological expansion, thus creating a certain distribution of wormholes in the Friedman space. Moreover, the inflationary stage in the past [18] should enormously stretch characteristic scales of the relic foam. The foam-like structure, in turn, was shown to be flexible enough to account for the all the variety of DM phenomena [10, 14]; for parameters of the foam may arbitrary vary in space to produce the observed variety of DM halos in galaxies (e.g., the universal rotation curve for spirals constructed by [13] for the foamed Universe perfectly fits observations). Moreover, the topological origin of DM phenomena means that the DM halos surrounding point-like sources appear due to the scattering on topological defects and if a source radiates, such a halo turns out to be luminous too [16] which seems to be the only way to explain naturally the observed absence of DM fraction in intracluster gas clouds [9].

The foam-like structure of the Universe is represented by the gas of wormholes randomly distributed in space. It was demonstrated recently by [11] that in the presence of such a gas every point source turns out to be surrounded with a "dark halo" which possesses both signs depending on scales and the background distribution of wormholes. Therefore, it seems to be not quite clear for readers whether wormholes produce the necessary CDM picture, or they merely suggest some additional effects. Moreover, there still exists some wide-spread mistaken opinion that wormholes lead to non-Gaussian perturbations in the analogy with

other words the fact that the distribution of DM should have cusps in galaxies is equivalent to the fact that DM should represent cold non-baryonic particles.

topological defects of another kind (strings, monopoles, etc.). In the present paper we clarify such problems by means of considering the development of density perturbations in the gas of wormholes. We demonstrate that at very large scales wormholes behave exactly like very heavy particles and thus reproducing all the predictions of CDM. They however predict an additional specific damping in the development of baryon perturbations. Moreover, at smaller scales the non-linear stage of the evolution of perturbations essentially diverges from that in CDM. At small scales there exists a rather strong non-gravitational interaction between particles and wormholes (due to the mutual scattering) which surely cure the problem of cusps in galaxies.

The complete analysis of the gravitational dynamics of wormholes is rather complicated. Therefore, in the present paper we restrict ourself with the Newton approximation to derive basic equations which govern the dynamics of baryons and wormholes. While the problem of the generalization to the General Relativity we leave for the future research.

The paper is organized as follows. In Sec.2 we introduce a wormhole and describe it's general properties. In Sec. 3 we show that in the Friedman model wormholes acquire non-vanishing rest masses. In Sec. 4 we describe the process of the scattering of particles on a wormhole. In Sec. 5 we infer the motion equations for particles and wormholes in the Newtonian approximation for the expanding reference system. In Sec.6 we introduce the system of Boltzmann - Vlasov equations which describes kinetics of particles, wormholes, and collisions. In Sec. 7 we infer the non-relativistic hydrodynamic equations with corrections for the collisions between particles and wormholes. In Sec.8 we calculate basic kinetic coefficients which describe the collisions. In Sec.9 we consider the behavior of linear perturbations in the Newtonian approximation. In particular, we explicitly demonstrate that at large distances wormholes behaves exactly like heavy non-baryon particles reproducing thus the standard CDM picture. While the strong coupling at smaller scales cures the basic failure of CDM (i.e., it removes cusps in galaxies). In the last section we discuss results obtained and show further perspectives.

2 Wormholes

The simplest wormhole is described by the metric

$$ds^2 = c^2 dt^2 - h^2(r) \delta_{\alpha\beta} dx^\alpha dx^\beta, \quad (1)$$

where

$$h(r) = 1 + \theta(b-r) \left(\frac{b^2}{r^2} - 1 \right) \quad (2)$$

and $\theta(x)$ is the step function ($\theta(x) = 0$ as $x < 0$ and $\theta(x) = 1$ as $x > 0$). We point out the obvious properties $h'_r = \theta(b-r) \left(\frac{b^2}{r^2} \right)'$ and $h''_{rr} = \delta(b-r) \left(\frac{b^2}{r^2} \right)' (b-r)' + \theta(b-r) \left(\frac{b^2}{r^2} \right)''$. Such a wormhole has vanishing throat

length. Indeed, for the region $r > b$ $h = 1$ and the metric coincides merely with that in the Minkowsky space, while the region $r < b$, upon the obvious transformations $y^\alpha = \frac{b^2}{r^2} x^\alpha$, gives the same region $y > b$ with the same flat metric $ds^2 = c^2 dt^2 - \delta_{\alpha\beta} dy^\alpha dy^\beta$. Therefore, both regions $r > b$ and $r < b$ represent similar portions of two Minkowsky spaces glued at the surface of the sphere S^3 with the center at the origin $r = 0$ and the radius $r = b$. Thus, formally, such a space can be described with the ordinary double-valued flat metric in the region $r_\pm > b$ as

$$ds^2 = c^2 dt^2 - \delta_{\alpha\beta} dx_\pm^\alpha dx_\pm^\beta, \quad (3)$$

where the sign \pm in coordinates x_\pm^α stands to describe two different sheets of space.

A generalization appears when we change the step function $\theta(x)$ with any smooth function $\tilde{\theta}(x)$ which has the same property $\tilde{\theta}(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $\tilde{\theta}(x) \rightarrow 1$ as $x \rightarrow 0$. On the contrary to the previous case such a wormhole will have a non-vanishing throat length. However in the last case the consideration differs in details, while general features remain the same. We also point out that such wormholes have vanishing mass. In general, one may also insert a non-vanishing mass to the wormhole as it is described in [19].

First of all we consider the stress energy tensor which produces such a wormhole. It can be found from the Einstein equation $T_\alpha^\beta = R_\alpha^\beta - \frac{1}{2}\delta_\alpha^\beta R$. Since the metric (1) does not depend on time we find

$$R_0^0 = R_\alpha^0 = 0, \quad R_\alpha^\beta = -P_\alpha^\beta = -\frac{2}{b}\delta(b-r)\{n_\alpha n^\beta - \delta_\alpha^\beta\} \quad (4)$$

where $n^\alpha = n_\alpha = x^\alpha/r$ is the outer normal to the sphere S^3 and therefore

$$T_\mu^\nu = -\frac{2}{b}\delta(b-r)(\delta_\mu^\nu - \Delta_\mu^\nu) \quad (5)$$

where $\Delta_0^0 = \Delta_\alpha^0 = 0$, and $\Delta_\alpha^\beta = \delta_\alpha^\beta - n_\alpha n^\beta = \sum_{A=1,2} e_\alpha^A e^{A\beta}$ where e_α^A are two unite tangent to the sphere vectors $e_\alpha^A n^\alpha = 0$. Thus, the effective source can be considered as a mixture of a negative "cosmological constant" $T_\mu^{0\nu} = -\frac{2}{b}\delta(b-r)\delta_\mu^\nu$ and two components of a perfect fluid $T_\mu^{A\nu} = -\frac{2}{b}\delta(b-r)u_\mu^A u^{A\nu}$ with zero pressure $p = 0$, energy density $\varepsilon = -\frac{2}{b}\delta(b-r)$, and the velocity $u^{A\nu} = (0, e^{A\beta})$. The velocities have purely space-like (tangent to the surface of the sphere) components $u_\mu^A u^{A\mu} = -1$. This matter is concentrated only on the surface of the sphere due to the multiplier $\delta(b-r)$. All such sources represent an exotic matter which cannot be constructed from actual particles. Recall, that all real particles have time-like velocities $u_\mu u^\mu = 1$ or isotropic $u_\mu u^\mu = 0$ (if the rest mass vanishes $m = 0$). However, this property is violated for virtual particles. This gives us a hint that vacuum polarization effects can in principle be collected to organize such a form of matter. By other words to construct a wormhole we have to organize a negative cosmological term on the surface S^3 and then to rotate the surface with the space-like velocity in the two different directions. From the other hand the presence of a wormhole leads to the vacuum

polarization and therefore to the exotic matter (e.g., see [20, 21]). Here we leave aside the important problem of what precedes the egg (wormhole) or the hen (exotic matter).

The inner and outer regions of the sphere S^3 are equal, which gives the possibility to construct a wormhole which connects regions in the same space (instead of two independent spaces). This is achieved by the identification (gluing) in (3) of the two spaces by means of the use of possible motions of the flat space. Let \vec{R}_+ be the position of the sphere in coordinates x_+^α , then the gluing is the rule

$$x_+^\alpha = R_+^\alpha + U_\beta^\alpha (x_-^\beta - R_-^\beta) \quad (6)$$

where $U_\beta^\alpha \in O(3)$, which represents the composition of a translation and a rotation of the flat space. In terms of common coordinates such a wormhole represents the standard flat space in which the two spheres S_\pm^3 (with centers at positions R_\pm^α) are glued by the rule (6)

If we neglect the higher order images with respect to the transformations $\xi_\pm^\alpha = \frac{b^2}{\xi_\pm^2} \xi_\pm^\alpha$ (which are the reflections with respect to spheres S_\pm^3), then we can use the metric (1) in which we merely replace h with the function

$$h(r, R_\pm) = 1 + \theta(b - |\xi_+|) \left(\frac{b^2}{\xi_-^2} - 1 \right) + \theta(b - |\xi_-|) \left(\frac{b^2}{\xi_+^2} - 1 \right) \quad (7)$$

where $\xi_\pm^\alpha = x^\alpha - R_\pm^\alpha$ and $\xi_\pm'^\alpha = (U_\beta^\alpha)^{\pm 1} \xi_\pm^\beta$. In this case the physically admissible region of space is the outer region of the two spheres S_\pm^3 , while the inner regions represent only additional images of the outer space and have no meaning².

Thus, we see that the simplest wormhole may be described by a set of parameters $\eta = (R_+^\alpha, R_-^\alpha, b, U_\beta^\alpha)$. We point out that in the general case a wormhole possesses a rather complex properties, e.g., parameters possess a dynamics $\eta = \eta(t)$ and, moreover, (b, U_β^α) are functions of coordinates x^α which describe the specific structure of the wormhole.

The generalization to a set of wormholes is given by

$$h(r, R_\pm) = 1 + \sum_{n, \sigma=\pm} \theta(b_n - |\xi_{n\sigma}|) \left(\frac{b_n^2}{\xi_{n-\sigma}^2} - 1 \right) \quad (8)$$

and the identification of points at joint spheres $S_{n\pm}^3$ as $\xi_{n+}^\alpha = U_{n\beta}^\alpha \xi_{n-}^\beta$.

3 The rest mass of a wormhole in the expanding Universe

As it was demonstrated previously (e.g., see for details [11]) the presence of a gas of wormholes leads to a specific topological polarizability of space. This can

²If we do not neglect the higher order images, then within every sphere will appear a countable set of spheres $S_{A\pm}^3$ which represent additional images (copies) of the outer (physically admissible) region of space.

be described as a bias of point-like sources of gravity

$$\delta(r - r_0) \rightarrow \delta(r - r_0) + B(r, r_0),$$

where the bias gives the polarization mass generated on throats. The bias consists of two terms $B = B_0 + B_1$, where B_0 resembles the bias of spherical mirrors and gives the positive contribution (anti-screening) to the total mass of a particle

$$B_0(r) = \sum_{n, \sigma = \pm} \frac{b_n}{R_\sigma} \left[\delta(\vec{r} - \vec{r}_{n, \sigma}) - \delta(\vec{r} - \vec{R}_{n, \sigma}) \right], \quad (9)$$

where $\vec{r}_{n, \pm} = \vec{R}_{n, \pm} + \frac{a^2}{(\vec{r}_0 - \vec{R}_\mp)^2} U_n^{\pm 1} (\vec{r}_0 - \vec{R}_{n, \mp})$, while the rest part gives pure screening and is given by

$$B_1(r) = \sum_n b_n \left(\frac{1}{R_{n, +}} - \frac{1}{R_{n, -}} \right) [\delta(\vec{r} - \vec{r}_{n, -}) - \delta(\vec{r} - \vec{r}_{n, +})]. \quad (10)$$

In the homogeneous Universe (i.e., in the case of a homogeneous distribution of matter) the second part disappears automatically, while the first part generates the rest mass for every wormhole. Indeed, to demonstrate this we rewrite the bias (9) in the form (see for details [11])

$$B_0(r) = \frac{\partial h(\vec{r})}{\partial r^\alpha} \frac{\partial(-1/r)}{\partial r^\alpha} + 4\pi h(0) \delta(\vec{r}), \quad (11)$$

where

$$h(r) = \int \frac{r^\alpha R^\beta}{R^2} \left[H_{\alpha\beta}^+ (\vec{r}, \vec{R}) + H_{\beta\alpha}^+ (\vec{R}, \vec{r}) \right] d^3 R,$$

$$H_{\alpha\beta}^\pm(R_+, R_-) = \int b^3 U_{\alpha\beta}^{\pm 1} F(R_\pm, b, U) da dU,$$

and $F(\eta)$ is the distribution of wormholes in the configuration space. In the case of the homogeneous and isotropic distribution of wormholes, averaging over the rotation matrix gives $H_{\alpha\beta}^\pm(R_+, R_-) = \frac{1}{3} \delta_{\alpha\beta} \phi(|R_+ - R_-|)$, where $\phi(X) = \int b^3 F(X, b) db$. Therefore, the bias reduces to the form

$$B_0(r, r_0) = \frac{4\pi}{3} n \overline{b^3} \delta(\vec{r} - \vec{r}_0) \quad (12)$$

where n is the density of throats, while $\frac{4\pi}{3} n \overline{b^3}$ is the portion of the unit volume which is cut by throats. In the above formula the only difference from spherical mirrors (the multiplier 1/3) appears due to averaging over the rotation matrix. By other words every wormhole increases the mass of a point-like source in proportion to the volume which the wormhole cuts from the space. When the space is filled with homogeneously distributed matter, every wormhole acquires the rest mass $M_w(b) = \frac{4\pi}{3} b^3 \rho$ (ρ is the mean density).

All the above consideration can be omitted if we note that the wormhole possesses straightforward generalization to the case of the expanding Universe. Indeed, the metric is

$$ds^2 = a^2(\tau) (d\tau^2 - h^2(r) \delta_{\alpha\beta} dx^\alpha dx^\beta). \quad (13)$$

In this case the wormhole expands with the space and acquires a non-vanishing rest mass

$$M_w = \frac{4}{3} \pi a^3 b^3 \rho_b \quad (14)$$

where ρ_b is the mean density of matter in the Universe. By other words the gluing procedure cuts two equal homogeneous portions of space (the two spheres S_\pm^3) filled with a homogeneously distributed matter. Thus, the total amount of matter diminishes, while spheres acquire the rest masses which compensate the distribution of the effective density of matter to the homogeneity³. We point out that the metric (13), with (8) taken into account, directly shows that an arbitrary primordial distribution of wormholes in space agrees with the visible homogeneity of space (the Friedman model) as it was first stated by [10]. An inhomogeneous distribution of wormholes in space (e.g., fractal distribution) leads to the dual fractal distribution of particles (for particles may occupy only the physically admissible region of space), while the total effective density remains perfectly homogeneous.

From the astrophysical standpoint such a mass is much smaller than that of a typical object (e.g., if the throat radius is $r_{th} = ab \sim R_0$, and $\rho_b = \rho_{cr}$, where R_0 is the Sun radius and ρ_{cr} is the critical density, then the mass has the order $M_w \sim 10kg$). Thus, gravitational effects of such objects in Solar systems is merely negligible. However in the dynamics of larger systems (galaxies, clusters, etc.) they become the more and more visible, e.g., as the presence of dark matter. We point out that such dark particles (wormholes) are extremely heavy from the particle physics point of view. It is clear that the picture where the Universe is filled with a gas of "particles" whose rest masses have the order $\sim 10kg$ perfectly fits the basic Cold Dark Matter model (CDM) widely accepted today. And as it is well known, predictions of CDM perfectly agree with observations at very large (clusters of galaxies and higher) scales. However, while the standard CDM (heavy non-baryon particles) makes a wrong prediction at smaller scales (e.g., it predicts cusps $\rho_{DM} \sim 1/r$ in centers of galaxies, when observations demonstrate the cored $\rho_{DM} \sim const$ distribution), we may expect that wormholes will cure such a problem. Indeed, since the two conjugated spheres S_\pm^3 (throats) represent merely the "same" region of space we may state that the local density at S_+^3 coincides exactly with that at S_-^3 . Let d_0 be the typical distance between throats. Thus, if one throat gets into the central region of a galaxy while the rest throat is sufficiently far from the center, then the total density will be somewhat smoothed. And the typical scale of the smoothing

³We also point out that rotating wormholes will be described by the tensor of inertia $I_{\alpha\beta}^w = I \delta_{\alpha\beta}$, with $I = \frac{2}{5} a^2 b^2 M_w$.

(i.e. the minimal core radius) will be of the order of d_0 . Of course, the rigorous consideration of this problem requires considering the proper dynamics of wormholes which we start in the next section.

4 The collision with a wormhole

Consider the simplest construction of a wormhole in space as follows. Let us fold the space at the plane $z = 0$. Then we can use the simplest metric (1) which connects the two half-spaces $z > 0$ and $z < 0$. Let the velocity of the wormhole is \vec{V} which, due to the above symmetry (i.e., $Z_+ = -Z_-$, where $\vec{R}_\pm = (X_\pm, Y_\pm, Z_\pm)$ are the positions of throats), corresponds to the case when one throat (say S_-^3) moves with the velocity $\vec{V}_- = \vec{V}$, while the other S_+^3 moves with $\vec{V}_+ = (V_x, V_y, -V_z)$. For the sake of simplicity we consider everywhere the non-relativistic case (i.e., $V \ll c$). In the case of a general wormhole the relation between \vec{V}_+ and \vec{V}_- is given by the same relation (6)

$$V_+^\alpha = U_\beta^\alpha V_-^\beta. \quad (15)$$

By other words since both spheres represent the same region of space their velocities are rigidly connected (the visible doubling of the number of degrees of freedom related to wormholes is fictitious).

Consider an incident on S_-^3 particle with the rest mass m and the initial velocity \vec{v} . Then the scattering of the particle leads to the transformation⁴

$$n_-^\alpha \rightarrow n_+^\alpha = U_\beta^\alpha n_-^\beta, \quad \vec{R}_\pm \rightarrow \vec{R}'_\pm = \vec{R}_\pm \quad (16)$$

where $n_\pm^\alpha = (x^\alpha - R_\pm^\alpha)/b$ are points at spheres S_\pm^3 , and the transformation of velocities

$$\vec{V}'_\pm = \vec{V}_\pm + \frac{2m}{M_w + m} (u_\pm n_\pm) \vec{n}_\pm, \quad (17)$$

$$\begin{aligned} v'^\alpha &= U_\beta^\alpha \left(v^\beta - \frac{2M_w}{M_w + m} (u_- n_-) n_-^\beta \right) = \\ &= V_+^\alpha + u_+^\alpha - \frac{2M_w}{M_w + m} (u_+ n_+) n_+^\alpha. \end{aligned} \quad (18)$$

Here $(un) = (\vec{u}\vec{n})$ denotes the ordinary scalar product and

$$\vec{u}_- = \vec{v} - \vec{V}_-, \quad u_+^\alpha = U_\beta^\alpha u_-^\beta,$$

and the rotational matrix $U_\beta^\alpha \in O(3)$ corresponds to the above symmetry (i.e., the reflection with respect to the plane $z = 0$, $z' = -z$).

⁴This law of the transformation can be easily obtained as follows. We notice that the metric (1) merely connects two equal flat spaces. Then the scattering of a particle merely coincides with the elastic scattering on a solid ball with simultaneous transport of the particle from one sheet of space to the other sheet.

We point out that the above law of transformation (16)-(18) holds in the most general case (the case of an arbitrary gluing, i.e., an arbitrary $U_\beta^\alpha \in O(3)$) as well. In the limit $M_w \rightarrow \infty$ this law has been considered by us in [15].

It is easy to see that the law of transformation (16)-(18) conserves the total energy

$$mv^2 + MV_-^2 = mv'^2 + MV_+^2$$

where the kinetic energy of the wormhole is accounted only for once. It maybe convenient to consider throats as an independent objects. Then, due to the rigid relation (15) we have to introduce the bound energy $\varepsilon = -\frac{1}{2}MV_-^2 = -\frac{1}{2}MV_+^2$ which gives the conservation of the total energy in the form

$$\frac{mv^2}{2} + \frac{MV_-^2}{2} + \frac{MV_+^2}{2} + \varepsilon = \frac{mv'^2}{2} + \frac{MV_-'^2}{2} + \frac{MV_+'^2}{2} + \varepsilon'. \quad (19)$$

It is important that if we forget about the factorization (i.e., the identification of points or the gluing) (6), then the above transformation does not conserve the momentum. Instead we get the conservation in the form

$$MV_+^{\prime\alpha} + mv'^\alpha = U_\beta^\alpha (MV_-^\alpha + mv^\alpha). \quad (20)$$

It happens due to the fact that the space with the wormhole does not possess the translation symmetry. It possesses however the somewhat entangled (with the gluing (6) taken into account) symmetry.

We point out that when we consider dynamics of wormholes the matrix U_β^α becomes function on time (formation of gravitationally bounded objects leads naturally to origin of rotational motions).

5 Equations of motion

As it was shown above a general wormhole possesses a rest mass and, therefore, it should move in space as an ordinary test particle. For the sake of simplicity we shall use an approximation when wormholes can be considered as point-like objects, i.e., we neglect the size of wormholes. For cosmological applications this represents a rather good approximation, while for some astrophysical problems it can be not sufficient.

The presence of a gas wormholes in the Universe (i.e. of the complexity of the topological structure of space) leads to an enormous complexity in consideration of the dynamics (due to boundary conditions at wormholes). To simplify the problem we shall use the Newton's equations, while the generalization to the relativistic case we leave for the future. Moreover, as it is well known (e.g., see [22]) the Newton's approach is acceptable for a rather huge range of scales. It will be also convenient to introduce the expanding reference system from the very beginning.

5.1 Newton's equation

When the gravitational field is rather weak, the Einstein equations reduce to

$$\nabla_r^2 \Phi = 4\pi G \left(\rho + \frac{3P}{c^2} \right) \quad (21)$$

where $\nabla_r^2 \Phi \approx R_{00}$ is the time component of the Ricci tensor. The equations of motion for a test particle are

$$\frac{d^2 r^\alpha}{dt^2} = -\Phi_{,\alpha} \quad (22)$$

which have the range of the applicability $v \ll c$, or (equivalently) at distances $R \ll cH^{-1}$ ($H = \dot{a}/a$ is the Hubble constant). The lower boundary is given by the gravitational radius for compact objects (black holes).

Consider the homogeneous component $P(t)$ and $\rho(t)$ and the homogeneous expansion of space $\vec{r} = a(t)\vec{x}$. Then the equation (21) gives

$$\Phi_b = \frac{2}{3}\pi G \left(\rho_b(t) + \frac{3P_b(t)}{c^2} \right) r^2,$$

while the equations of motion (22) lead to the standard cosmological equation

$$\frac{d^2 a}{dt^2} = -\frac{4}{3}\pi G \left(\rho_b(t) + \frac{3P_b(t)}{c^2} \right) a. \quad (23)$$

Here the point $x = 0$ corresponds to the observer (which gives $\Phi_b \sim r^2$) and the applicability is restricted by

$$g_{00} = 1 + 2\Phi/c^2 \quad 2\Phi \ll c^2$$

which means that distances r cannot be too large.

5.2 Peculiar velocity

Consider now the expanding reference system $\vec{r} = a(t)\vec{x}$. Then the total velocity is

$$\vec{u} = a\dot{\vec{x}} + \vec{x}\dot{a} = a\dot{\vec{x}} + H\vec{r}$$

where $a\dot{\vec{x}}$ is the peculiar velocity, while $H\vec{r}$ gives the standard Hubble expansion.

For a particle the equations of motion can be obtained from the Lagrangian ([22])

$$\mathcal{L} = \frac{1}{2}m \left(a\dot{\vec{x}} + \vec{x}\dot{a} \right)^2 - m\Phi(\vec{x}, t).$$

Let us use the canonical transformation

$$\mathcal{L} \rightarrow \mathcal{L} - \frac{d\psi}{dt}, \psi = \frac{1}{2}m\dot{a}a^2x^2$$

which transforms the Lagrangian function to the form

$$\mathcal{L} = \frac{1}{2}ma^2\dot{\vec{x}}^2 - m\varphi,$$

where

$$\varphi = \Phi + \frac{1}{2}a\ddot{a}x^2$$

and new Newtonian potential φ satisfy the equation

$$\frac{1}{a^2}\nabla^2\varphi = 4\pi G\left(\rho + \frac{3P}{c^2}\right) + 3\frac{\ddot{a}}{a}$$

where the gradient is taken over \vec{x} . Then by means of use of the cosmological equation (23) it transforms the Poisson equation to the form

$$\frac{1}{a^2}\nabla^2\varphi = 4\pi G\left(\delta\rho(\vec{x},t) + \frac{3\delta P(\vec{x},t)}{c^2}\right),$$

where $\delta\rho = \rho(\vec{x},t) - \rho_b(t)$. The equations of motions for a test particle become

$$\vec{p} = ma^2\dot{\vec{x}}, \quad \frac{d\vec{p}}{dt} = -m\vec{\nabla}\varphi \quad (24)$$

where the peculiar velocity $\vec{v} = a\dot{\vec{x}}$ relates to the momentum as $\vec{v} = \vec{p}/ma$ and by virtue of (24) obeys the equation

$$\frac{d\vec{v}}{dt} + \frac{\dot{a}}{a}\vec{v} = -\frac{1}{a}\vec{\nabla}\varphi.$$

E.g., if $\vec{\nabla}\varphi = 0$, then $\vec{p} = \text{const}$ and the velocity changes as $\vec{v} \sim \frac{\vec{p}}{ma}$, i.e., $\sim 1/a(t)$.

Thus, to describe the cosmological model filled with a set of particles and wormholes we get the system of equations as follows:

$$\begin{aligned} \ddot{a} &= -\frac{4}{3}\pi G\left(\rho_b(t) + \frac{3P_b(t)}{c^2}\right)a \\ \Delta\varphi &= 4\pi G\left(\delta\rho(\vec{x},t) + \frac{3\delta P(\vec{x},t)}{c^2}\right)a^2 \\ m_A a^2 \dot{\vec{x}}_A &= p_A \\ \dot{\vec{p}}_A &= -m_A \vec{\nabla}\varphi(x_A) \end{aligned} \quad (25)$$

where $A = 1, \dots, N$ numerates particles or wormholes.

This system of equations looks as the ordinary one but we should take into account that wormholes insert some complex (depending on time) identification of points (boundary conditions for φ and \vec{x}_A). Moreover, since wormholes are described by a larger number of parameters than a particle, i.e., $\eta_A, \dot{\eta}_A$ ($\eta = (R_+, R_-, b, U_\beta^\alpha)$) to complete this system we should add equations which govern the evolution of the rest parameters (b_A and $U_{\beta A}^\alpha$). For the sake of simplicity in the present paper we shall neglect the possible evolution of b and U_β^α .

6 Boltzmann -Vlasov type equations

In what follows we shall use the standard methods (e.g., see [23]). Let us introduce the density of particles in the phase space as $\Gamma_m = (\vec{x}, \vec{p})$,

$$dN_m = f_m(\Gamma_m, t) d\Gamma_m, \quad d\Gamma_m = d^3x d^3p$$

and the number density of wormholes

$$dN_w(\gamma) = F_w(\Gamma_w, \gamma, t) d\Gamma_w.$$

here, since we assume the part of parameters $\gamma = (b, U_\beta^\alpha)$ be fixed (non-dynamical ones), $d\Gamma_w = d\eta dp_\eta$, $\Gamma_w = (\eta, p_\eta)$, and, $\eta = (\vec{R}_+, \vec{R}_-)$. We also note that according to (15) only one momentum is free. It is convenient to consider both momenta $p_\eta = (P_+, P_-)$ as independent, while the relation (15) is accounted for by a delta-function - type multiplier in F_w . Then the matter density can be expressed via $f_m(\vec{x}, \vec{p}, t)$ as

$$\rho_m(\vec{x}, t) = \frac{m}{a^3} \int \delta(\vec{x} - \vec{x}') f_m(\Gamma'_m, t) d\Gamma'_m = \rho_b(t) [1 + \delta_m(\vec{x}, t)] \quad (26)$$

and analogously the contribution from wormholes is (here $M(\gamma) = \frac{4}{3}\pi b^3 a^3 \rho_b$)

$$\rho_w(\vec{x}, t) = \sum_{\sigma=\pm} \int \frac{M(\gamma)}{a^3} \delta(\vec{x} - \vec{R}_\sigma) F_w(\Gamma_w, \gamma, t) d\Gamma_w d\gamma.$$

We point out that in the proper (expanding) reference system the mean number of particles in the volume element $(\Delta x)^3$ remains constant, i.e., $a^3 \rho_b \sim \text{const}$ and so do the masses of wormholes $M(\gamma) = M(b) \sim \text{const}$.

In $F_w(\Gamma_w, \gamma, t)$ indexes w and γ numerate merely the sort of wormholes. It is convenient to introduce the reduced distribution function for wormhole throats as follows

$$f_w(\vec{X}, \vec{P}, \gamma, t) = \sum_{\sigma=\pm} \int \delta(\vec{X} - \vec{R}_\sigma) \delta(\vec{P} - \vec{P}_\sigma) F_w(\Gamma_w, t) d\Gamma_w$$

which defines the mass density in the form analogous that of particles

$$\rho_w(\vec{x}, t) = \int \frac{M(\gamma)}{a^3} f_w(\vec{x}, \vec{P}, \gamma, t) d^3\vec{P} d\gamma = \rho_b(t) [1 + \delta_w(\vec{x}, t)]. \quad (27)$$

Then the system of Boltzmann-Vlasov equations are eqs. (26), (27),

$$\frac{\partial f_A}{\partial t} + \frac{\vec{p}}{m_A a^2} \nabla f_A - m_A \nabla \varphi \frac{\partial f_A}{\partial \vec{p}} = \sum_B st f_{AB}, \quad (28)$$

and the generalized Poisson equation (we set $\delta P/c^2 = \frac{c_s^2}{c^2} \delta \rho \ll \delta \rho$ otherwise one has to add the coefficient $(1 + 3c_s^2/c^2)$ before $\delta \rho$)

$$\frac{1}{a^2} \Delta \varphi = 4\pi G \left(\delta \rho(\vec{x}, t) + \int B(x, x') \delta \rho' d^3 \vec{x}' \right). \quad (29)$$

In the last equation the bias $B(x, x')$ accounts for the topological polarizability of space in the presence of the gas of wormholes (i.e., the proper boundary conditions at wormhole throats). As it was shown by [11] it expresses completely via $F_w(\Gamma_w, \gamma, t)$ and is given by (9), (10). In (28) the index A denotes either m in the case of particles, or (w, γ) in the case of wormhole throats.

Rigorously speaking the above system is not complete, since the collisions between particles and a wormhole involve both throats

$$stf_{AB} = \int w(\Gamma'_A, \Gamma'_B, \Gamma_A, \Gamma_B) [f'_A F'_B - f_A F_B] d\Gamma'_A d\Gamma'_B d\Gamma_B \quad (30)$$

where the scattering matrix $w(\Gamma'_A, \Gamma'_B, \Gamma_A, \Gamma_B)$ stands for the transformation law $(\Gamma_A, \Gamma_B) \rightarrow (\Gamma'_A, \Gamma'_B)$ which is defined by (16)-(18). The function F_A obeys the equation similar to the two-point correlation function

$$\frac{\partial F_A}{\partial t} + \sum_{\sigma=\pm} \left(\frac{\vec{p}_\sigma}{m_A a^2} \vec{\nabla}_\sigma - m_A \vec{\nabla}_\sigma \varphi \frac{\partial}{\partial \vec{p}_\sigma} \right) F_A = \sum_B stf_{AB}. \quad (31)$$

Then equations (28) are obtained by the integration of the last equation over the extra variables.

7 Fluid equations

Upon integrating (28) over $\int d^3p$ we get

$$\frac{\partial}{\partial t} \frac{\rho_A a^3}{m_A} + \frac{1}{a} \vec{\nabla} \left(\frac{\rho_A a^3}{m_A} \vec{u}_A \right) = D_A, \quad (32)$$

which represents the first hydrodynamic equation (the discontinuity equation)). The term $D_A = \int stf_A d^3p$ we define latter on, while we used the notions for the density

$$\int f_A d^3p = \frac{\rho_A(\vec{x}, t) a^3}{m_A},$$

the peculiar macroscopic velocity $\vec{u}(\vec{x}, t)$

$$\vec{u}_A(\vec{x}, t) = \frac{1}{m_A a} \frac{\int \vec{p} f_A d^3\vec{p}}{\int f_A d^3\vec{p}}, \quad (33)$$

and $\rho_A(\vec{x}, t) = \rho_b(t)(1 + \delta_A(\vec{x}, t))$. Due to the property $\rho_b a^3 = \text{const}$ this equation can be rewritten as

$$\frac{\partial}{\partial t} \delta_A(\vec{x}, t) + \frac{1}{a} \nabla_\beta \left((1 + \delta_A) u_A^\beta \right) = \frac{m_A}{\rho_b a^3} D_A. \quad (34)$$

We point out that in the standard hydrodynamics the terms of the type D_A vanish due to the local conservation laws (of the number of particles, momentum, energy). In collisions of particles and wormholes such terms retain. During the

scattering of particles on wormholes the number of throats conserves locally which gives $D_w = 0$. Though, in general, there remain processes of throats merging which produce some value $D_w < 0$. It is also easy to see that all such terms vanish in the absence of perturbations (for the homogeneous background).

Multiplying the equation (28) with p_α and integrating over momenta d^3p we get the second (Euler) equation

$$\frac{\partial}{\partial t} (a^4 \rho_A u_A^\alpha) + \rho_A a^3 \nabla_\alpha \varphi + a^3 \nabla_\beta (u_A^\alpha u_A^\beta \rho_A) + a^3 \nabla_\alpha P = Q_A^\alpha \quad (35)$$

where

$$Q_A^\alpha = \int p^\alpha s t f_A d^3p$$

and we used the obvious definitions

$$\langle v^\alpha v^\beta \rangle = \frac{\int p^\alpha p^\beta f d^3p}{m^2 a^2 \int f d^3p}, \quad (36)$$

$$\langle v^\alpha v^\beta \rangle = u_A^\alpha u_A^\beta + \frac{1}{3} \delta_{\alpha\beta} \langle \delta v^2 \rangle = u_A^\alpha u_A^\beta + \frac{P_A}{\rho_A} \delta_{\alpha\beta},$$

where P_A is the pressure. The system of equations (32) and (35) represents the basic hydrodynamic equations. In the same way one can also add to the above system the equation for the heat transport (i.e., the next momentum for the internal energy $m_A \langle \delta v^2 \rangle / 2$). For the sake of simplicity we do not consider the heat equation here. For adiabatic linear perturbations such equation is not important. It however becomes important more latter on non-linear stages of the formation of astrophysical objects.

In view of $\rho_b a^3 = \text{const}$ (35) maybe rewritten as ($H = \dot{a}/a$ is the Hubble constant)

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + H \right) ((1 + \delta_A) u_A^\alpha) + \frac{1}{a} (1 + \delta_A) \nabla_\alpha \varphi + \\ & + \frac{1}{a} \nabla_\beta ((1 + \delta_A) u_A^\alpha u_A^\beta) + \frac{1}{a \rho_b} \nabla_\alpha P_A = \frac{1}{a^4 \rho_b} Q_A^\alpha. \end{aligned}$$

Let us take the divergence from the last equation and in view of (34) we find the master equations which govern the evolution of density perturbations (here $Q_A = \nabla_\alpha Q_A^\alpha$)

$$\begin{aligned} & \left(\frac{\partial}{\partial t} + 2H \right) \left(\frac{\partial}{\partial t} \delta_A - \frac{m}{\rho_b a^3} D_A \right) = \frac{1}{a^2} \nabla_\alpha (1 + \delta_A) \nabla_\alpha \varphi + \\ & + \frac{1}{a^2} \nabla_\alpha \nabla_\beta ((1 + \delta_A) u_A^\alpha u_A^\beta) + \frac{1}{a^2 \rho_b} \nabla^2 P_A - \frac{1}{a^5 \rho_b} Q_A. \end{aligned}$$

Retaining just linear terms we get the equation for linear perturbations as

$$\left(\frac{\partial}{\partial t} + 2H \right) \left(\frac{\partial}{\partial t} \delta_A - \frac{m}{\rho_b a^3} D_A \right) = \frac{1}{a^2} \Delta (\varphi + c_{A,s}^2 \delta_A) - \frac{1}{a^5 \rho_b} Q_A \quad (37)$$

where we used the relation $\delta P = c_s^2 \delta \rho$ and c_s is the sound speed. We also have to add the Poisson equation

$$\frac{1}{a^2} \Delta \varphi = 4\pi G \rho_b \sum_A \left(\delta_A + \int B(x, x') \delta'_A d^3 x' \right). \quad (38)$$

Thus, the system (37) and (38) represents the master equation which describes the development of linear adiabatic perturbations in the presence of the gas of wormholes. This system represents the standard system for CDM model where wormholes play the role of dark matter particles. The difference appears however due to the additional terms D_A , Q_A , and the bias $B(x, x')$. For homogeneous background some terms disappear. As we shall see latter on they disappear in the long-wave limit as well i.e., as $k \ll k_J$ (where k_J is the Jeans wavelength whose definition in the presence of wormholes somewhat differs from the standard one).

8 Kinetic coefficients D and Q

Consider an arbitrary point \vec{x} on the sphere S_- , i.e., $\vec{x} \in S_-$ and therefore $\xi_-^2 = (\vec{x} - \vec{R}_-)^2 = b_w^2$. The gluing procedure transforms this point into a conjugated point $\vec{x}' \in S_+$ which has the form $\vec{x}' = \vec{R}_+ + \vec{\xi}_+$ where $\vec{\xi}_+$ relates to $\vec{\xi}_-$ by some rotation $\xi_+^\alpha = U_\beta^\alpha \xi_-^\beta$. Let $\Gamma_w = (R_\pm, b_w, U, V_\pm)$ denote the set of parameters of the wormhole. The scattering matrix W is presented as $W = W_+ + W_-$ where W_\pm corresponds to which of throats absorbs particles. Due to the obvious symmetry between throats this gives merely the factor 2 in final expressions. Then we find from (16)-(18) $W_\pm(\Gamma_m, \Gamma_w, \Gamma'_m, \Gamma'_w) = |u_\pm| \sigma_\pm(\Gamma_m, \Gamma_w, \Gamma'_m, \Gamma'_w)$ where

$$\begin{aligned} \sigma_- = & \delta(\xi_+ - b_w) \delta(\Gamma'_m - \Gamma_{m,-}) \delta(\Gamma'_w - \tilde{\Gamma}_w) - \\ & - \delta(\xi_- - b_w) \delta(\Gamma'_m - \Gamma_m) \delta(\Gamma'_w - \Gamma_w) \end{aligned} \quad (39)$$

and analogous term σ_+ with the obvious replacement $(- \rightarrow +$ and $U \rightarrow U^{-1})$, where we used the notions as follows $\tilde{\xi}_\pm = \vec{x} - \vec{R}_\pm$, $\vec{n}_\pm = \tilde{\xi}_\pm / b_w$,

$$\Gamma_{m,-} = (x_-, p_-), \quad \tilde{\Gamma}_w = (R_\pm, V'_\pm, b_w, U),$$

and the relations

$$\begin{aligned} x_-^\alpha &= R_+^\alpha + U_\beta^\alpha \xi_-^\beta, \\ \vec{p}_- &= U \left(\vec{p} - \frac{2M_w m a}{M_w + m} (u_- n_-) \vec{n}_- \right), \end{aligned} \quad (40)$$

$$\vec{V}'_\pm = \vec{V}_\pm + \frac{2m}{M_w + m} (u_\pm n_\pm) \vec{n}_\pm, \quad (41)$$

and

$$\vec{u}_- = \vec{v} - \vec{V}_-, \quad u_+^\alpha = U_\beta^\alpha u_-^\beta.$$

For astrophysical needs wormholes may be considered as point-like objects. This is achieved by the replacement in (39)

$$\delta(\xi_{\pm} - b_w) \rightarrow 4\pi b_w^2 \delta(\vec{R}_{\pm} - \vec{x}) \quad (42)$$

where $\delta(\vec{x})$ is the 3-dimensional delta function $\int \delta(\vec{x}) d^3x = 1$. Thus the above expressions completely define the scattering matrix W and the scattering term

$$stf_{AB} = \int W(\Gamma'_A, \Gamma'_B, \Gamma_A, \Gamma_B) [f'_A F'_B - f_A F_B] d\Gamma'_A d\Gamma'_B d\Gamma_B. \quad (43)$$

8.1 The scattering of particles

Consider first the terms D_m and Q_m . Since we suppose that $M_w \gg m$, it follows that $\langle V_{\pm}^2 \rangle \sim T/(aM_w) \ll \langle v^2 \rangle \sim T/(am)$ and to the leading order we can neglect the motions of wormholes which essentially simplifies the above expressions (e.g., $u_- = v = p/ma$ in (39) and (40)). We also suppose the homogeneous distribution of wormholes in space which gives $f_w(\eta) = \int f_w(\Gamma_w) dP_{\eta} = F(|\vec{R}_- - \vec{R}_+|, \gamma)$ (here $\gamma = b_w, U$). Then for the scattering term (30) we find

$$stf_{m,w} = 8\pi \int |v| [f'_m(R, p_-) - f_m(x, p)] b_w^2 F(|x - R|, \gamma) d\mu,$$

where $d\mu = d^3p d^3R d\gamma$. This defines the kinetic coefficient D_m for particles as (we use here the property $|v| = |v'|$ and $d^3p = d^3p'$)

$$D_m = \frac{8\pi}{ma} \int [|p'| f'_m(R, p') - |p| f_m(x, p)] g(|x - R|) d^3R d^3p,$$

where $g(R) = \int b_w^2 F(R, \gamma) d\gamma$ which gives

$$D_m = \frac{8\pi}{ma} \int [K(R+x) - K(x)] g(R) d^3R \quad (44)$$

with

$$K(x, t) = \int |p| f(x, p, t) d^3p. \quad (45)$$

We point out that for the homogeneous distribution of particles $K(x) = K = \text{const}$ and $D_m \equiv 0$.

Consider now the second coefficient Q_m^{α} which is given by

$$Q_m^{\alpha} = \frac{8\pi}{ma} \int \left[\frac{1}{9} K_{\alpha}(R+x) - K_{\alpha}(x) \right] g(R) d^3R, \quad (46)$$

where

$$K_{\alpha}(x) = \int p_{\alpha} |p| f(x, p) d^3p. \quad (47)$$

The multiplier $1/9$ appears in (46) upon transformations as follows. According to (40)

$$\begin{aligned} K'_\alpha(x) &= \int [p_\alpha |p'| f(x, p')] d^3 p' = \\ &= U_{\alpha\beta} \int (p'_\beta - 2(p'n)n_\beta) |p'| f(x, p') d^3 p'. \end{aligned}$$

The replacement (42) means an additional averaging over n_β which gives $\langle n_\alpha n_\beta \rangle = \frac{1}{3} \delta_{\alpha\beta}$, while assuming an isotropic distribution of wormholes over the rotation matrix U gives an additional multiplier $\langle U_{\alpha\beta} \rangle = \frac{1}{3} \delta_{\alpha\beta}$. Thus, we see that $K'_\alpha(x) = \frac{1}{9} K_\alpha(x)$ in (46). The above expressions (44) and (46) show that the kinetic coefficients are expressed via the momenta $K(x)$ and $K_\alpha(x)$.

8.2 functions $K(x)$ and $K_\alpha(x)$

Keeping in mind the linearized equations (37) we will use the following form for the quasi-equilibrium distribution function

$$f_0(x, p) = \frac{n(x, t)}{(2\pi maT(x, t))^{3/2}} \exp\left(-\frac{(p - mau(x, t))^2}{2maT(x, t)}\right)$$

which corresponds to the locally thermodynamic equilibrium distribution and $n(x, t) = \rho(\vec{x}, t)a^3/m$. Then we find

$$K(n, u, T) = \frac{n}{(2\pi maT)^{3/2}} \int \exp\left(-\frac{(\vec{p} - ma\vec{u})^2}{2maT}\right) |p| d^3 p.$$

Expanding this by \vec{u} (i.e., $K = K^0 + K^1_\alpha u_\alpha + \frac{1}{2} K^2_{\alpha\beta} u_\alpha u_\beta + \dots$) we find

$$K = n \frac{2}{\sqrt{\pi}} (2maT)^{1/2} \left(1 + \frac{ma}{6T} u^2\right) + \dots \quad (48)$$

To evaluate the function $K_\alpha(x)$ we point out the obvious relations

$$\frac{\partial}{\partial u_\alpha} f_0(x, p) = -\frac{mau_\alpha - p_\alpha}{T} f_0(x, p)$$

which defines the function $K_\alpha(x)$ in the form

$$K_\alpha(x) = T \left(\frac{\partial}{\partial u_\alpha} + \frac{mau_\alpha}{T} \right) K$$

and from (48) we find

$$K_\alpha(x) \simeq 2Tn \sqrt{\frac{2maT}{\pi}} \left(\frac{\partial}{\partial u_\alpha} + \frac{mau_\alpha}{T} \right) \left(1 + \frac{ma}{6T} u^2\right).$$

Thus, in the leading order by \vec{u} we get

$$K \simeq \frac{2n}{\sqrt{\pi}}(2maT)^{1/2}, \quad K_\alpha(x, t) \simeq \frac{4}{3}K(x, t) mau^\alpha(x, t).$$

Using the relations

$$\langle \delta v^2 \rangle = \frac{3P(\vec{x}, t)}{\rho(\vec{x}, t)} = \frac{1}{m^2 a^2} \frac{\int p^2 f d^3 p}{\int f d^3 p} = \frac{3maT}{m^2 a^2}$$

or $T = maP/\rho$, we may rewrite the function K in the equivalent form

$$K(x, t) \simeq \frac{2\sqrt{2}}{\sqrt{\pi}} \rho a^4 c_s \left(\frac{P}{\rho c_s^2} \right)^{1/2}.$$

For linear perturbations we shall use $\delta P_m = c_s^2 \delta \rho_m = c_s^2 \rho_b(t) \delta_m$ which gives

$$\delta K(x, t) \simeq \frac{1}{2} K(t) \left(\frac{\rho}{P} c_s^2 + 1 \right) \frac{\delta \rho}{\rho}, \quad (49)$$

and, therefore, we get the functions we are looking for in the form

$$\delta K \simeq \frac{1}{2} K(t) \left(1 + c_s^2 \frac{\rho_b}{P} \right) \delta(\vec{x}, t), \quad \delta K_\alpha \simeq \frac{4}{3} K(t) mau_\alpha(x, t). \quad (50)$$

8.3 Kinetic coefficients D_m and Q_m

In what follows we shall use the parameter $P_m/\rho_m c_s^2 \simeq 1$ which gives $\delta K \simeq K(t) \delta(\vec{x}, t)$ and $K \simeq \frac{2\sqrt{2}}{\sqrt{\pi}} \rho a^4 c_s$. Then by the use of the above formulas we finally find from (44) the kinetic coefficients as

$$D_m(x, t) = \frac{8\pi}{ma} K \int [\delta_m(x + R, t) - \delta_m(x, t)] g(R) d^3 R \quad (51)$$

and analogously from (46)

$$Q_m = \frac{32\pi}{3} K \int \left[\frac{1}{9} \nabla_\alpha u_m^\alpha(x + R, t) - \nabla_\alpha u_m^\alpha(x, t) \right] g(R) d^3 R. \quad (52)$$

In the last equation the term $\nabla_\alpha u_m^\alpha$ can be expressed from (34) as

$$\nabla_\alpha u_m^\alpha = -a \left(\frac{\partial}{\partial t} \delta_m - \frac{m}{\rho_b a^3} D_m \right),$$

while for the quantity $\frac{m}{\rho_b a^3} D_m$ we find

$$\frac{m}{\rho_b a^3} D_m \simeq 16\sqrt{2\pi} c_s \int [\delta(x + R, t) - \delta(x, t)] g(R) d^3 R.$$

Consider now some qualitative estimates. Let $\int g(R) d^3R = \langle b_w^2 \rangle n_w$ (n_w is the density of wormholes in the comoving reference system, the physical density is n_w/a^3). Then we may estimate

$$\frac{m}{\rho_b a^3} D_m \sim 16\sqrt{2\pi} \frac{1}{\tau_w} \frac{d_w}{L} \delta \sim \nu_w \delta$$

where $\tau_w \sim (c_s b_w^2 n_w)^{-1}$, d is the characteristic distance between throats, and L is the characteristic scale of the inhomogeneity. For estimates we also can suppose $n_w = \rho_w a^3 / M_w$, where $M_w = \rho_b \frac{4}{3} \pi a^3 b_w^3$ which gives

$$\tau_w \sim \frac{M_w}{c_s b_w^2 \rho_w a^3} = \frac{4}{3} \pi \frac{b_w}{c_s} \frac{\rho_b}{\rho_w}$$

and finally we define the estimate for the collision frequency

$$\nu_w(k) \sim \frac{6\sqrt{2}}{\pi\sqrt{\pi}} \frac{d_w}{b_w} \frac{\rho_w}{\rho_b} k c_s.$$

It can be seen that for $k c_s \rightarrow 0$ this correction is negligible (the collision frequency $\nu_w \rightarrow 0$), while for rather short wave-length this term may give the leading contribution.

8.4 Kinetic coefficients D_w and Q_w

As it was already pointed out the collisions conserve the local number of throats which immediately gives the value

$$D_w = 0. \tag{53}$$

Some non-vanishing value of $D_w < 0$ may appear when we take into account for the processes of throats merging (annihilation of wormholes). Such processes are important on the very early stages of the evolution of the Universe, since they are responsible for the formation of the background distribution of wormholes in space. E.g., the origin of the Tully-Fisher relation for spirals was suggested by [13] which requires the decay for some portion of primordial wormholes. Such a decay is accompanied with an essential reheating, since every merging of a wormhole throats radiates the energy $\sim M_w c^2$. We however leave this problem aside for the future research.

In the case of the isotropic background the second coefficient Q_w does not require a separate evaluation. In spite of the fact that during the scattering the momentum does not conserve (20), some kind of the conservation law for mean values takes however place. Indeed, due to the obvious symmetries $S_+^3 \leftrightarrow S_-^3$ and $t \rightarrow -t$

$$w(\Gamma'_A, \Gamma'_B, \Gamma_A, \Gamma_B) = w(\Gamma_A, \Gamma_B, \Gamma'_A, \Gamma'_B)$$

we find that

$$Q_{m,\alpha} = \langle p_\alpha \rangle = \frac{1}{2} \langle p_\alpha - p'_\alpha \rangle,$$

where $\langle p_\alpha \rangle = \int p_\alpha st f_{m,w} d^3 p$. And analogous expression holds for throats $Q_{w,\alpha} = \langle P_\pm \rangle = \frac{1}{2} \langle P_\pm - P'_\pm \rangle = \int P_{\pm\alpha} st f_{w,m} d^3 P_\pm$. Due to the symmetry $S_+^3 \rightarrow S_-^3$ we can always change $\langle P_+ \rangle = \langle P_- \rangle$. Then using the relation (20) we find

$$\frac{1}{2} \langle p - p' \rangle + \frac{1}{2} \langle P_- - P'_+ \rangle = \frac{1}{2} \langle (1 - U) p \rangle + \frac{1}{2} \langle (1 - U) P_- \rangle.$$

For the isotropic distribution the averaging over the matrix U gives $\langle U_{\alpha\beta} \rangle = \frac{1}{3} \delta_{\alpha\beta}$ and, therefore, the above expression transforms to $Q_m = -Q_w + \frac{1}{3} Q_m + \frac{1}{3} Q_w$, i.e.,

$$Q_m = -Q_w. \quad (54)$$

The last equation means not more than the conservation of the total mean momentum density. It expresses the balance for the transformation of the momentum between particles and wormholes.

9 The behavior of linear perturbations

Consider now the Fourier transform for the density perturbations

$$\delta_A(x) = (2\pi)^{-3/2} \int e^{i\vec{k}\vec{x}} \delta_{A,\vec{k}} d^3 k.$$

Then we find

$$\frac{m}{\rho_b a^3} D_A(k) = -\nu_A(k) \delta_{A,\vec{k}} \quad (55)$$

where

$$\nu_m(k) = (8\pi)^2 c_s \left[g(0) - g(-\vec{k}) \right], \quad \nu_w(k) = 0 \quad (56)$$

Analogously we find

$$\frac{1}{a^5 \rho_b} Q_m(k) = -\frac{1}{a^5 \rho_b} Q_w(k) = \Omega_k \left(\frac{\partial}{\partial t} + \nu_m(k) \right) \delta_{m,\vec{k}} \quad (57)$$

where

$$\Omega_k = \frac{4}{3} (8\pi)^2 c_s \left[g(0) - \frac{1}{9} g(-\vec{k}) \right]. \quad (58)$$

Since we assume that $M_w \gg m$, with a very good approximation we may set $\delta P_w = 0$. Therefore, the master equations are

$$\left(\frac{\partial}{\partial t} + 2H + \Omega_k \right) \left(\frac{\partial}{\partial t} + \nu_m(k) \right) \delta_{m,\vec{k}} + \frac{k^2 c_s^2}{a^2} \delta_{m,\vec{k}} + \frac{k^2}{a^2} \varphi_k = 0 \quad (59)$$

$$\left(\frac{\partial}{\partial t} + 2H \right) \frac{\partial}{\partial t} \delta_{w,\vec{k}} - \Omega_k \left(\frac{\partial}{\partial t} + \nu_m(k) \right) \delta_{m,\vec{k}} + \frac{k^2}{a^2} \varphi_k = 0 \quad (60)$$

and the Poisson equation (for the Newton potential)

$$-k^2 \frac{1}{a^2} \varphi_k = 4\pi G (1 + B(k)) \rho_b \left(\delta_{m,\vec{k}} + \delta_{w,\vec{k}} \right), \quad (61)$$

where the function $B(k)$ was determined by [11] and has the form⁵

$$B(k) = \frac{8\pi(g_1(k) - g_1(0))}{k^2}, \quad (62)$$

where $g_1(X) = \int b_w F(R, \gamma) d\gamma$ (so that $\int g_1(x) d^3x = \langle b_w \rangle n_w$). We point out that all the coefficients Ω_k , $\nu_m(k)$, and $B(k)$ are functions of the background (quasi-stationary) distribution of wormholes $F(|R_+ - R_-|, \gamma)$ whose exact form requires an independent further investigation.

9.1 Perturbations in the density of wormholes $\delta_{w,\vec{k}}$

Consider first the case when $\delta_{m,\vec{k}} = 0$. Then the equation (60) with (61) taken into account reads

$$\left(\frac{\partial}{\partial t} + 2H\right) \frac{\partial}{\partial t} \delta_{w,\vec{k}} - 4\pi G_k \rho_b \delta_{w,\vec{k}} = 0.$$

where $G_k = G(1 + B(k))$. We point out that this equation coincides with the standard equation for perturbations in cold dark matter particles. Some difference appears however due to the presence of the bias $B(k)$ which reflects the polarizability of space filled with the gas of wormholes. Formally such a polarizability looks like a scale-dependent renormalization of the gravitational constant. By other words, at very large scales (and on the linear stage of the development of perturbations) the gas of wormholes reproduces exactly the dark matter particles.

The essential difference appears however on the non-linear stage due to the existence of the mutual exchange with the momentum between wormholes and baryons (54) (57). It is clear that such an exchange cure the basic failure of CDM (i.e., the presence of cusps in the center of galaxies), since dark halos around galaxies should rotate.

9.2 Matter perturbations $\delta_{m,\vec{k}}$

If we set $\delta_{w,\vec{k}} = 0$, then the master equation for perturbations becomes

$$\left(\frac{\partial}{\partial t} + 2H + \Omega_k\right) \left(\frac{\partial}{\partial t} + \nu_m(k)\right) \delta + \left[\frac{k^2 c_s^2}{a^2} - 4\pi G_k \rho_b\right] \delta = 0. \quad (63)$$

The additional coefficients $\nu_m(k)$ and Ω_k have the clear physical interpretation. $\nu_m(k)$ is the collision frequency which describes the processes of the absorption and re-radiation of particles by wormholes. It is clear that such processes somewhat smooth the initial inhomogeneity in the particle number density which results in the specific damping we already discussed in [15]. At very large distances $k \gg d$ (where d is the characteristic distance between throats) this kind

⁵We recall that the bias B consists of the two terms (9) and (10). For perturbations the leading contribution comes from B_1 , while the first part B_0 is accounted for by the insertion of the rest masses of wormholes.

of damping vanishes $\nu_m(k) \rightarrow 0$. The additional friction term, proportional to Ω_k , describes the mutual interchange with momentum between particles and wormholes. We point out that in the linear approximation there exists only a transport of the momentum density from particles to wormholes which also leads to an additional damping of inhomogeneities. Moreover, this kind of damping retains in the long-wave limit as well ($\Omega_0 \neq 0$).

Consider the redefinition

$$\delta_{\vec{k}} = \exp\left(-\int^t \nu(k) dt\right) \tilde{\delta}_{\vec{k}}$$

then, for the new quantity $\tilde{\delta}_{\vec{k}}$ we find the standard-type equation

$$\frac{\partial^2}{\partial t^2} \tilde{\delta}_{\vec{k}} + 2H_k \frac{\partial}{\partial t} \tilde{\delta}_{\vec{k}} + \left[\frac{k^2 c_s^2}{a^2} - 4\pi G_k \rho_b \right] \tilde{\delta}_{\vec{k}} = 0 \quad (64)$$

where

$$2H_k = 2H + \Omega_k - \nu(k), \text{ and } G_k = G(1 + B(k)).$$

By other words, this equation formally coincides with the standard one with the renormalized friction H_k and the gravitational G_k constants. The presence of the additional damping $\nu(k)$ and the friction Ω_k somewhat weakens the standard Jeans instability. Indeed, in the absence of the expansion ($H = 0$) (64) defines the dispersion relations ($\delta_{\vec{k}} \sim \exp(\lambda t)$)

$$(\lambda + \nu)^2 + 2H_k(\lambda + \nu) + \left[\frac{k^2 c_s^2}{a^2} - 4\pi G_k \rho_b \right] = 0$$

which defines the two (decaying and increasing) modes $\delta_{\vec{k}}^{1,2} \sim \exp(\lambda_{1,2} t)$ as

$$\lambda_{1,2} = -\frac{(\Omega_k + \nu(k))}{2} \pm \sqrt{4\pi G_k \rho_b + \frac{(\Omega_k - \nu(k))^2}{4} - \frac{k^2 c_s^2}{a^2}}.$$

In the limit $k \rightarrow 0$ we find for the rate of the growth

$$\lambda_1 \simeq \sqrt{4\pi G_0 \rho_b + \frac{\Omega_0^2}{4}} - \frac{\Omega_0}{2}.$$

9.3 Estimates for the background distribution of wormholes

As it follows from (55) - (57), (62) the functions $\nu(k)$, Ω_k , and $B(k)$ depend on the background distribution of wormholes in space. The background distribution requires an independent consideration and we present it elsewhere, while in this section we present some qualitative consideration.

As it can be seen from (31) the function F_w obeys the equation similar to the two-point correlation function. We consider here the two different limits

(the correct answer lies somewhere in the middle). First, we consider the frozen wormholes (i.e., $V_{\pm} = 0$). Then the function $F_w = F(|R_- - R_+|, b_w, U)$ is completely determined by the process of the wormhole production during the quantum stage (presumably an inflationary) of the evolution of the Universe. Let us choose the simplest case when $F_w \sim \delta(|R_- - R_+| - d_w)$ (e.g., see [11]). Then we get

$$g(R) = \frac{\bar{b}_w^2 n_w}{4\pi d_w^2} \delta(R - d_w), \quad g_1(R) = \frac{\bar{b}_w n_w}{4\pi d_w^2} \delta(R - d_w)$$

where n_w is the density of throats, \bar{b}_w is the mean throat radius, and d_w is the distance between throats. Therefore, we find $g(k) = \bar{b}_w^2 n_w (2\pi)^{-3/2} \sin(kd_w)/(kd_w)$ which defines the functions $\nu(k)$, Ω_k , and $B(k)$ in the form

$$\begin{aligned} B(k) &= -2n_w \bar{b}_w (2\pi)^{-1/2} \frac{1}{k^2} \left(1 - \frac{\sin(kd_w)}{kd_w}\right), \\ \nu_m(k) &= (8\pi)^2 c_s \bar{b}_w^2 n_w (2\pi)^{-3/2} \left(1 - \frac{\sin(kd_w)}{kd_w}\right), \\ \Omega_k &= \frac{4}{3} (8\pi)^2 c_s \bar{b}_w^2 n_w (2\pi)^{-3/2} \left[1 - \frac{1}{9} \frac{\sin(kd_w)}{kd_w}\right]. \end{aligned}$$

which for $kd_w \ll 1$ give

$$\begin{aligned} B(k) &\approx -\frac{2n_w \bar{b}_w}{(2\pi)^{1/2}} \frac{1}{6} d_w^2 \left(1 - \frac{1}{20} (kd_w)^2 + \dots\right), \\ \nu_m(k) &\approx \frac{(8\pi)^2}{6} c_s \bar{b}_w^2 n_w (2\pi)^{-3/2} k^2 d_w^2, \\ \Omega_k &\approx \frac{4}{3} \frac{(8\pi)^2}{9} c_s \bar{b}_w^2 n_w (2\pi)^{-3/2} \left(8 + \frac{1}{6} k^2 d_w^2 + \dots\right). \end{aligned}$$

Thus, we see that in the long-wave limit $kd_w \ll 1$ we have the behavior $\nu_m \sim k^2$, while $\Omega, B \sim \text{const.}$

We point out that this case can presumably be far from reality, since the presence of hot matter makes wormholes move chaotically. What we should expect that $F_w(V) \sim \exp(-M_w V^2/2T)$ has the Maxwell-like form. However, due to the enormous typical value of the rest mass $M_w \sim 10^{28} m_p$, the approximation of frozen wormholes can in turn work sufficiently well.

The second case corresponds to the opposite situation, when all correlations between conjugated throats are lost, i.e., when $F_w = F(|R_- - R_+|) \sim \text{const.}$ Then we find $g(k) \sim g_0 \delta(k)$ and all quantities $\nu(k)$, Ω_k , and $B(k)$ tend to their asymptotic (as $k \rightarrow 0$) values.

10 Summary

In the present section we collect basic results. First of all we showed that the background density generates a non-vanishing rest mass of a wormhole (14). Therefore, wormholes may play the role of dark matter particles in CDM models. Then, we considered the scattering between a particle and a wormhole which straightforwardly shows that wormholes move in space. We derived basic equations (16)-(18) in the non-relativistic case ($V \ll c$) which however admit straightforward generalization to the relativistic case. Based on the scattering equations we suggested the kinetic description in Sec. 6 and derived the basic fluid equations in Sec. 7. All those equations are generalized straightforwardly to the relativistic case as well and we present it elsewhere. In Sec. 9 we derived equations for linear density perturbations and have shown that at large scales density perturbations, related to wormholes, behave exactly like standard dark matter particles. However, even on the linear stage there always exists the transport of the momentum density from baryons to wormholes. Already such a phenomenon is enough to cure the basic failure of CDM particles, namely, to remove the cusps in centers of galaxies. Thus, we suppose that at present wormholes represent the best candidate for dark matter particles. We also demonstrated that development of perturbations in baryons possesses an additional damping (with respect to the standard Newtonian instability). However the complete analysis requires considering the relativistic case which we present elsewhere.

As far as basic equations are concern, the generalization to the relativistic (GR) case of our basic results seem to have not essential difficulties (Boltzmann, fluid equations). The basic difficulty appears when considering the generalization for the bias operator $B(x, x')$ which reflects the polarizability of space. For scalar wave equation in the geometric optics approximation such bias was considered recently by [16]. However such an approximation works only on sufficiently late stage of the evolution of the Universe (where the Newtonian consideration works well), while for very early stage it is not sufficient. This poses a rather serious problem for future research.

We point also out that for small astrophysical objects (e.g., jets, nuclei of galaxies, etc.) one probably has to account for the dynamics of the additional parameters of wormholes (b_w, U) which should essentially complexify the theory presented.

The gas of wormholes represents an extremely reach (by physical effects) but complex medium. Due to the existence of the polarizability (which has the topological origin) it has all properties of a gravitational plasma. It possesses also numerous additional non-trivial phenomena and we think it worth saying that we have deal with a new state of matter.

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References

- [1] Zwicky F., 1933, *Helv. Phys. Acta*, 6, 110; Zwicky F., 1954, *Morphological Astronomy*, Springer-Verlag, Berlin
- [2] Borner G., 1992, *The Early Universe: Facts and Fiction*, Springer-Verlag, New York; Peacock J.A., 1998, *Cosmological Physics*, Cambridge University Press, Cambridge.
- [3] Primack J.R., 2001, astro-ph/0112255, Lectures at International School of Space Science, L'Aquila, Italy, August-September.
- [4] Navarro, J.F., Frenk, C.S., White, S.D.M., 1996, *ApJ*, 462, 563.
- [5] Diemand J., et.al., 2005, *Mon. Not. Roy. Astron. Soc.* 364, 665.
- [6] Gentile, G., Salucci, P., Klein, U., Vergani, D., Kalberla, P., 2004, *MNRAS*, 351, 903; Weldrake, D.T.F., de Blok, W. J. G., Walter, F., 2003, *MNRAS*, 340, 12; de Blok, W. J. G., Bosma, A., 2002, *A&A*, 385, 816
- [7] Finzi A., 1963, *MNRAS*, 127, 21; Tohline J.E., 1983), In: *The Internal Kinematics and Dynamics of Galaxies*, IAU Symp. 100, ed. E Athanassoula, pp. 205-206. Dordrecht: Reidel; Kuhn J.R., Kruglyak L., 1987, *Ap.J*, 313, 1-12; Eckhardt D.H., 1993, *Phys.Rev. D* 48, 3762-3767; Hadjimichef D., Kokubun F., 1997, *Phys.Rev.*, D 55, 733-738; Drummond I.T., 2001, *Phys.Rev. D*, 63, 1.
- [8] Bekenstein J.D., M. Milgrom, 1984, *Ap. J.* , 286, 7; Bekenstein J.D., 1993, *Phys.Rev. D*, 48, 3641-3647; Bekenstein J., 2006, *Contemporary Physics* 47, 387
- [9] Clowe D., et al., 2006, *Astrophys. J. Lett.*, 648, L109
- [10] Kirillov, A.A., 2006, *Phys. Lett. B*, 632, 453
- [11] Kirillov A.A., Savelova E.P., 2008, *Phys. Lett. B*, 660, 93-99
- [12] Kirillov A.A., Savelova E.P., 2008, *Gravitation and Cosmology*, 14, 256-261
- [13] Kirillov, A.A., Turaev, D., 2006, *MNRAS*, 371, L31
- [14] Kirillov A.A., Turaev D., 2007, *Phys. Lett. B*, 656, 1
- [15] Kirillov, A.A, Savelova E.P., Zolotarev P.S., 2008, *Phys. Lett. B*, 663, 372-376
- [16] Kirillov A.A., Savelova E.P., Shamshutdinova G.D., 2009, *JETP Lett.*, 90, 599-603
- [17] Wheeler J.A., 1964 in: *Relativity, Groups, and Topology*, B.S. and C.M. DeWitt (eds.), Gordon and Breach, New York; Hawking S.W., 1978, *Nuclear Phys. B*, 114, 349

- [18] Gliner E.B., 1965, Zh. Eksp. Teor. Fiz. 49, 542; Starobinsky A.A., 1980, Phys. Lett. B, 91, 100; Guth A.H., 1981, Phys. Rev. D, 23, 347; Linde A.A., 1982, Phys. Lett. B, 108, 389.
- [19] Visser M., 1996, Lorentzian wormholes, Springer-Verlag, New-York, Inc.
- [20] Grib A.A., Mamaev S.G., Mostepanenko V.M., 1988, Vacuum quantum effects in intensive fields, M: Energoatomizdat, [in russian].
- [21] Khatsymovsky V., 1994, Phys. Lett. B, 320, 234-240.
- [22] Peebles P.J.E., 1980, The large-scale structure of the Universe, Princeton Univ. Press, Princeton, NJ
- [23] Stewart J.M., 1971, *Non-Equilibrium Relativistic Kinetic Theory*, in Lecture Notes in Physics, Vol. 10, Springer, Berlin